Mock final

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Discrete Mathematics II, Winter 2011/12
Due date: Decmber 13th (Tuesday) by 12:30, at the beginning of the exercise session.

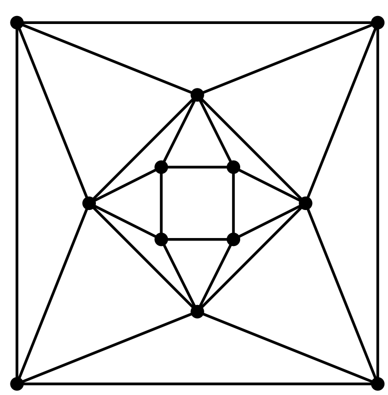
You can submit solutions to all problems. All of them will be properly graded, but will not be counted towards your homework requirements. You may choose to time yourself (2 hours), but you do not need to. Good solutions to five problems is usually enough for a 1,0. It is a good idea to review the material before you start solving the test.

Problem 1. Determine the Turán number of $K_{1,4}$ for every n.

Problem 2. Prove the theorem of Chvátal and Erdős: if $\kappa(G) \geq \alpha(G)$, then G has a Hamilton cycle.

Problem 3. Prove that every $(k-1)$-edge-connected k-regular graph on an even number of vertices has a perfect matching.

Problem 4. Define a sequence of plane graphs as follows. Let $G_1 = C_4$. For $n > 1$ obtain G_n from G_{n-1} by adding a new 4-cycle surrounding G_{n-1}, making each vertex of the new cycle also adjacent to two consecutive vertices of the previous outside face. The graph G_3 is shown below.
Prove that if n is even, then every proper 4-coloring of G_n uses each color on exactly n vertices.



Problem 5. Suppose that for an integer $k \in \mathbb{N}$ and simple graph G we have that $\delta(G) \geq k$ and $n(G) \geq 2k$. Prove that $\alpha'(G) \geq k$.

Problem 6. Show that the Petersen graph contains a $K_{3,3}$-minor.