email: person@math.fu-berlin.de

Material covered

Lecture 1, 10th April 2012 (M. Aigner, "A Course in Enumeration", chapter 1): Rule of Sum, Rule of Product, Rule of Bijection, Rule of Counting in two ways (double counting). Subsets and Binomial coefficients. Binomial theorem. Pascal recurrence, Multisets, number of k-multiset of an n-element set. Stirling numbers of the 2^{nd} kind and relation to the number of surjections.

Lecture 2, 16th April 2012 (M. Aigner, "A Course in Enumeration", chapter 1): Recurrence relation for Stirling numbers of the 2^{nd} kind. Number of mappings as a sum of the Striling numbers. Permutations (from S_n) and their cycle decomposition, Stirling numbers of the 1^{st} kind and the recurrence relation for them.

Lecture 3, 17th April 2012 (M. Aigner, "A Course in Enumeration", chapter 1): Small Stirling numbers of the 1^{st} kind. Ordered and unordered number-partitions. Summary: (distinguishable/nondistinguishable) balls into (distinguishable/nondistinguishable) boxes. Falling and rising factorials and extension of binomial coefficients to polynomials and complex numbers. Polynomial method. Multinomial coefficients and multinomial theorem.

Lecture 4, 23th April 2012 (M. Aigner, "A Course in Enumeration", chapter 1): Polynomial method. Some binomial identities, alternating partial sums of binomial coefficients. Polynomial relations for Stirling numbers of the 1^{st} and 2^{nd} kind. Obtaining recursion formula for Stirling numbers of the second kind: $S_{n,k} = S_{n-1,k-1} + kS_{n-1,k}$ from $x^n = \sum_{k=0}^{n} S_{n,k} x^{\underline{k}}$. Lattice paths.

Lecture 5, 24th April 2012 (M. Aigner, "A Course in Enumeration", chapter 1; J. Matoušek and J. Nešetřil, "Invitation to Discrete Mathematics", Chapter 3.7 and 3.8): Lattice paths and the binomial identity

$$\sum_{k=0}^{n} \binom{s+k}{k}\binom{n-k}{m} = \binom{s+n+1}{s+m+1} \quad \text{for } s,m,n \in \mathbb{N}_0.$$

Two proofs of the inclusion-exclusion (IE) principle. An inefficient secretary and derangements. Using IE principle for proving binomial identities with alternating signs.

Lecture 6, 30th April 2012 (J. Matoušek and J. Nešetřil, "Invitation to Discrete Mathematics", Chapter 3.7 and 3.8); R. A. Brualdi "Introductory Combinatorics", Chapter 2): Third proof of the inclusion-exclusion (IE) principle. Euler function φ. Pigeonhole principle: simple and strong forms. Applications of the simple form.

Lecture 7, 7th May 2012 (R. A. Brualdi "Introductory Combinatorics", Chapter 2; S. Jukna "Extremal Combinatorics", Chapter 4): Pigeonhole principle: simple and strong forms. Theorem of Dirichlet (rational approximation of irrational numbers), Theorem of Erdős and Szekeres (increasing vs. decreasing sequences). Graph (definitions, notation). Ramsey numbers (definition).

**Lecture 8, 8th May 2012 (S. Jukna "Extremal Combinatorics", Chapter 4; J. Matoušek and J. Nešetřil, "Invitation to Discrete Mathematics",

Chapter 12 **):** Ramsey numbers (definition). Upper (2^{2k-3}) and lower bounds ($2^{k/2}$) for the Ramsey number $R(k)$. Linear homogeneous recurrence relations.

Lecture 9, 14th May 2012 (R. A. Brualdi "Introductory Combinatorics", Chapter 7): Solving linear homogeneous recurrence relations. Fibonacci numbers. Vandermonde matrix. Characteristic polynomial: distinct roots vs. not necessarily distinct roots cases.

Lecture 10, 15th May 2012 (N. L. Biggs "Discrete Mathematics", Chapter 25; J. Matoušek and J. Nešetřil, "Invitation to Discrete Mathematics", Chapter 12**):** Rings $\mathbb{F}[x]$ and $\mathbb{F}[\![x]\!]$. Formal power series/ generating functions. Binomial theorem for negative exponents. Partial fractions. Fibonacci numbers revisited (via generating functions).

Lecture 11, 21st May 2012 (N. L. Biggs "Discrete Mathematics", Chapter 25; J. Matoušek and J. Nešetřil, "Invitation to Discrete Mathematics", Chapter 12**):** Partial fractions decomposition. The homogeneous linear recurrence revisited. Solving a nonhomogeneous linear recurrence relation (example) using generating functions. Analytic approach to the generating functions and the binomial theorem for real powers $\alpha \in \mathbb{R}$:

$$(1+x)^\alpha = \sum_{n=0}^{\infty} \binom{\alpha}{n} x^n.$$

Useful operations with sequences and its generating functions.

Lecture 12, 22nd May 2012 (R. A. Brualdi "Introductory Combinatorics", Chapter 7**; M. Aigner, "A Course in Enumeration", Chapter** 5**; H. Wilf Generatingfunctionology, Chapter** 2**)** Number of triangulations of a convex polygonal region with n sides. Catalan numbers. Exponential generating functions (derangements revisited). The involution principle.

Lecture 13, 29th May 2012 (M. Aigner, "A Course in Enumeration" Chapter 5**; R. Diestel Graph Theory, Chapter** 1**)** The involution principle: counting Catalan paths. Introduction to graph theory. Terminology, basic concepts: isomorphism, automorphism, properties, (induced) subgraphs, proper and spanning subgraphs, minimum, average and maximum degree, complete graphs K_n, paths with k edges P_k, cycles with k edges C_k, connectedness, distance, girth, circumference, complement of a graph. Handshaking lemma and its consequence for the number of vertices of odd degree in a graph. Graphs with average degree d contain subgraphs with minimum degree at least $\frac{1}{2}d$.

Lecture 14, 4th June 2012 (R. Diestel "Graph Theory", Chapter 1**)** Cycles and paths in graphs of minimum degree $\delta(G) \geq 2$. Euler tour theorem. Trails, walks, closed walks/trails. Trees and forests. Tree theorem. Proposition about enumerating vertices in a tree T by $v_1, \ldots, v_{|T|}$, such that v_i has exactly one neighbor among $\{v_1, \ldots, v_{i-1}\}$ for $i \geq 2$. Maximal/maximum and minimal/minimum with respect to some property.

Lecture 15, 5th June 2012 (R. Diestel "Graph Theory", Chapters 1 **and** 2**)** Bipartite, r-partite graphs. Characterization of bipartite graphs (no cycles of odd length). Matchings: maximal, maximum and perfect (1-factor). Alternating and augmenting paths. Vertex cover: minimal and minimum. The size of the maximum matching in G is denoted by $\alpha'(G)$, the cardinality of the minimum vertex cover is denoted by $\beta(G)$. Kőnig-Egerváry theorem in bipartite graphs ($\alpha'(G) = \beta(G)$).

Lecture 16, 11th June 2012 (R. Diestel "Graph Theory", Chapter 2) Hall's theorem (marriage theorem'1935). Regular graphs and k-factors. Corollaries of Hall's theorem: (1) every regular bipartite graph contains a perfect matching; (2) every regular graph with all vertex degrees being positive even numbers contains a 2-factor (Petersen 1891).

Lecture 17, 12th June 2012 (R. Diestel "Graph Theory", Chapter 2) Tutte's condition on the existence of perfect matchings in general graphs. Every bridgeless cubic graph contains a perfect matching (Petersen 1891).

Lecture 18, 18th June 2012 (R. Diestel "Graph Theory", Chapters 1 and 3) A-B-paths, independent paths, H-paths, separators, cutvertices, bridges. Connectivity number and k-connectedness ($\kappa(G)$). Edge-connectedness ($\lambda(G)$). Proposition: $\kappa(G) \leq \lambda(G) \leq \delta(G)$. Characterization of 2-connected graphs (ear decomposition theorem).

Lecture 19, 19th June 2012 (R. Diestel "Graph Theory", Chapter 5) Vertex and edge colorings. Chromatic number $\chi(G)$ and chromatic index $\chi'(G)$, k-colorability. Simple upper bounds on $\chi(G)$ in terms of edges of G, of maximum degree (greedy colorings) and coloring number. Theorem of Brooks.

Lecture 20, 25th June 2012 (R. Diestel "Graph Theory", Chapter 4 and 5) Theorem of Kőnig: $\chi'(G) = \Delta(G)$ for G bipartite. Theorem of Vizing (without proof): $\chi'(G) \in \{\Delta(G), \Delta(G)+1\}$. Planar graphs: introduction; arcs and polygonal arcs, drawings.

Lecture 21, 26th June 2012 (J. Matoušek and J. Nešetřil, "Invitation to Discrete Mathematics", Chapter 6) Planar drawings and graphs. Euler's formula for planar graphs ($v(G) - e(G) + f(G) = 2$). Estimates on the number of edges in a planar graph. Five Color Theorem. Hamiltonicity(introduction).

Lecture 22, 2nd July 2012 (D.B.West "Introduction to graph theory", Chapter 7; R. Diestel "Graph Theory", Chapter 10) Hamiltonicity(introduction). Theorems of Dirac and Ore. Theorem of Chvátal and Erdős. Labeled graphs - number of Hamilton paths and cycles on the vertex set $[n]$.

Lecture 23, 3rd July 2012 (D.B.West "Introduction to graph theory", Chapter 2;) Cayley's formula for the number of spanning trees in K_n. Prüfer code. Weighted graphs and minimum spanning trees (MST)- algorithm of Kruskal.

Lecture 24, 9th July 2012 (D.B.West "Introduction to graph theory", Chapter 2;) Weighted graphs and minimum spanning trees (MST)- algorithm of Kruskal (proof). Graph traversals: breadth-first search (BFS) and depth-first search (DSF) and the associated trees. Shortest paths in weighted graphs: Dijkstra's algorithm.

Lecture 25, 10th July 2012 (D.B.West "Introduction to graph theory", Chapter 2;) Shortest paths in weighted graphs: Dijkstra's algorithm and its modification to obtain the shortest paths tree from a fixed vertex v. Epilogue.