Diskrete Mathematik II WS 2012 / 2013
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Exercise Sheet 10

Due date: Jan 8, 12:30 PM, beginning of exercises
NO LATE SUBMISSIONS!

You should try to solve and write up all the exercises. You are welcome to submit **at most** two neatly written exercises each week. You are encouraged to submit in pairs, but don't forget to mark the name of the scriber.

Exercise 1.

Let $\mathbf{c}, \mathbf{d} \in \mathbb{R}^n$. Let further $A \in \mathbb{R}^{m \times n}$ and $\mathbf{b} \in \mathbb{R}^m$. Formulate a linear program that finds $\mathbf{x} \in \mathbb{R}^n$ such that

$$\frac{\mathbf{c}^T\mathbf{x}}{\mathbf{d}^T\mathbf{x}}$$

is maximised, subject to the constraints $A\mathbf{x} = \mathbf{b}$ and $\mathbf{x} \geq 0$. It is assumed that the set of feasible solutions $\mathbf{x}$ is **non-empty and bounded** in $\mathbf{R}^n$, and that for any feasible solution $\mathbf{x}$ the denominator $\mathbf{d}^T\mathbf{x}$ is **strictly positive**.

Exercise 2.

A point $\mathbf{x}$ is called *an extremal point* of a convex set $C \subseteq \mathbf{R}^n$ if $\mathbf{x} \in C$ and there are no two points $\mathbf{y}, \mathbf{z} \in C$ different from $\mathbf{x}$ such that $\mathbf{x}$ lies on the segment $\mathbf{yz}$.

Now let P be the set of all feasible solutions of a linear program in equational form (so P is a convex polyhedron). Show that $\mathbf{v} \in P$ is an extremal point if and only if it is a vertex of P.

(**Hint.** Show that if $\mathbf{v}$ is not a basic feasible solution, then it is possible to slightly alter its coordinates and find two other points in P such that $\mathbf{v}$ lies on the segment between them.)

Exercise 3.

Consider the following linear program.

$$\begin{aligned}
\text{maximize } & z = 4x_1 + 5x_2 \\
\text{subject to } & 2x_1 + x_2 \leq 6 \\
& x_1 + x_2 \leq 4 \\
& x_1 + 2x_2 \leq 6 \\
& x_1, x_2 \geq 0.
\end{aligned}$$

Solve this program using the simplex method. For the pivot step use Bland's rule: choose the improving variable with the smallest index, and if there are several possibilities for the leaving variable, choose the one with the smallest index. For each step write down the simplex tableau, the variable which enters the basis, and the variable which leaves the basis. Also write down the optimal feasible solution found by the algorithm and the value of the objective function for it.