

Exercise Sheet 11

You should try to solve and write up all the exercises. You are welcome to submit **at most** two neatly written exercises each week. You are encouraged to submit in pairs, but don't forget to mark the name of the scribe.

Exercise 1.

Consider the following linear program.

$$\begin{aligned} & \text{maximize } z = x_1 + x_2 \\ & \text{subject to } -2x_1 + x_2 \leq 2 \\ & \quad \quad \quad x_1 - 3x_2 \leq 6 \\ & \quad \quad \quad -x_1 - 2x_2 \leq -4 \\ & \quad \quad \quad x_1, x_2 \geq 0. \end{aligned}$$

(a) Express it in equational form.

(b) Define the auxiliary linear program for finding a feasible basis. Solve the auxiliary linear program using the simplex method and the Largest Coefficient pivot rule.

(**Hint:** For the auxiliary program, introduce just one new variable for the inequality with negative right-hand side).

(c) Use the feasible basis found at (b) to solve the initial program, using the simplex method and Bland's rule.

If the program is unbounded, provide a certificate (i.e. a line along which the objective function increases to infinity). If the program is infeasible, do not solve (c) and explain why.

For each step of the simplex algorithm write down the simplex tableau, the variable which enters the basis, and the variable which leaves the basis. Also write down the optimal solution found by the algorithm and the value of the objective function for it.

Exercise 2.

Consider the following linear program.

$$\begin{aligned} & \text{maximize } z = x_1 + x_2 \\ & \text{subject to } x_1 - 4x_2 \leq 3 \\ & \quad \quad \quad -x_1 - 2x_2 \leq -5 \\ & \quad \quad \quad 20x_2 - x_1 \leq 1 \\ & \quad \quad \quad x_1, x_2 \geq 0. \end{aligned}$$

(a) Express it in equational form.

(b) Define the auxiliary linear program for finding a feasible basis. Solve the auxiliary linear program using the simplex method and the Steepest Edge pivot rule.

(**Hint:** For the auxiliary program, introduce just one new variable for the inequality with negative right-hand side).

(c) Use the feasible basis found at (b) to solve the initial program, using the simplex method and Bland's rule.

If the program is unbounded, provide a certificate (i.e. a line along which the objective function increases to infinity). If the program is infeasible, do not solve (c) and explain why.

For each step of the simplex algorithm write down the simplex tableau, the variable which enters the basis, and the variable which leaves the basis. Also write down the optimal solution found by the algorithm and the value of the objective function for it.