Diskrete Mathematik II
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WS 2012 / 2013

Exercise Sheet 3

Due date: Nov 6, 12:30 PM, beginning of exercises
NO LATE SUBMISSIONS!

You should try to solve and write up all the exercises. You are welcome to submit **at most** two neatly written exercises each week. You are encouraged to submit in pairs, but don't forget to mark the name of the scriber.

Exercise 1.
Let $X := \{f, g, h, i, j\}$ and $Y := \{a, b, c, d, e\}$ and consider the complete bipartite graph G consisting of all edges between X and Y. We assign weights to the edges of G according to the following matrix, where the lines correspond to vertices in X, and the columns to vertices in Y:

$$\begin{array}{c c} & \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{array}{c} f \\ g \\ h \\ i \\ j \end{array} & \begin{pmatrix} 4 & 2 & 2 & 2 & 2 \\ 2 & 3 & 1 & 2 & 3 \\ 4 & 2 & 2 & 1 & 2 \\ 3 & 1 & 1 & 1 & 2 \\ 2 & 3 & 3 & 4 & 5 \end{pmatrix} \end{array}$$

Find a maximum weighted matching in G and a give a short proof that this matching has indeed maximum weight.

Exercise 2.
Formulate and prove a mathematically precise statement expressing the following: The stable matching produced by the Proposal Algorithm with the men doing the proposing is the best possible stable matching for the men.

Exercise 3. (a) Give an algorithm that finds the maximum and the minimum among n real numbers with at most $\lceil 3n/2 \rceil - 2$ comparisons.

(b) Prove that any algorithm needs that many comparisons in the worst case. (*Hint:* Try to devise a strategy to answer the queries of the algorithm in such a way, that it is forced to ask at least $\lceil 3n/2 \rceil - 2$ comparisons.)

Exercise 4. Let there be n bus drivers, n morning routes with durations $x_1, \ldots, x_n$, and n afternoon routes with durations $y_1, \ldots, y_n$. A driver is paid overtime, when the morning route and the afternoon route exceed the total time t. The objective is to assign one morning route and one afternoon route to each driver with minimum total amount of overtime. Express this as a weighted matching problem. Prove that giving the ith longest morning route and the ith shortest afternoon route to the same driver, for each i, does yield an optimal solution. (*Hint:* Do not use the Hungarian Algorithm, consider the special structure of the matrix.)