Diskrete Mathematik II

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WS 2012 / 2013

Exercise Sheet 4

Due date: Nov 13, 12:30 PM, beginning of exercises
NO LATE SUBMISSIONS!

You should try to solve and write up all the exercises. You are welcome to submit **at most** two neatly written exercises each week. You are encouraged to submit in pairs, but don't forget to mark the name of the scriber.

Exercise 1.

A total coloring of G assigns a color to each vertex and to each edge such that the colored objects have different colors when they are adjacent or incident. Prove that G has a total coloring with at most $\chi'_l(G) + 2$ colors.

Remark. The Total Coloring Conjecture states that every simple graph G has a total coloring with at most $\Delta(G) + 2$ colors. By the above exercise, the List Coloring Conjecture would imply a total coloring with $\Delta(G) + 3$ colors.

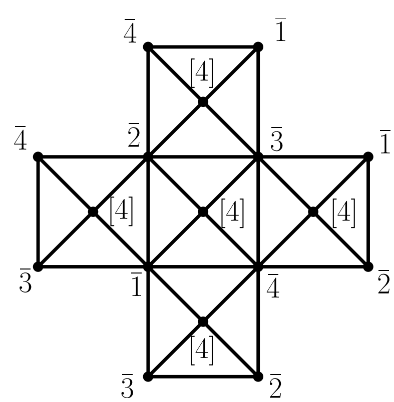
Exercise 2.

Prove that $K_{k,m}$ is k-choosable if and only if $m < k^k$.

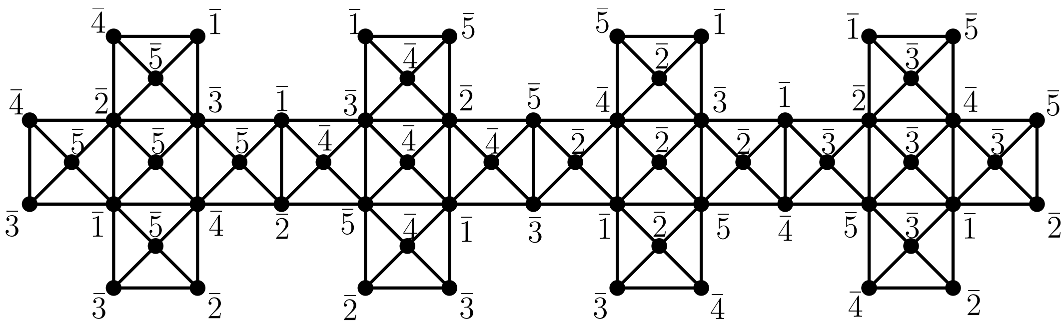
Exercise 3.

Show that the list-chromatic number of a planar graph can be as large as 5.

(a) Show that the following graph has no proper coloring from the assigned lists. (In this part of the exercise $\bar{i}$ denotes the set $[4] \setminus \{i\}$.)



(b) Let G be the graph obtained from the graph below by adding an extra vertex to the outside face and connecting it to all vertices on the boundary. Show that if the new vertex is assigned the list $\bar{1}$, then there is no proper list coloring of G from the assigned lists. (In this part of the exercise $\bar{i}$ denotes the set $[5] \setminus \{i\}$.)



Exercise 4.

Show that if G is a 3-regular graph then $\kappa(G) = \kappa'(G)$.