Diskrete Mathematik II WS 2012 / 2013
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Exercise Sheet 7

Due date: Dec 4, 12:30 PM, beginning of exercises
NO LATE SUBMISSIONS!

You should try to solve and write up all the exercises. You are welcome to submit **at most** two neatly written exercises each week. You are encouraged to submit in pairs, but don't forget to mark the name of the scriber.

Exercise 1^+ (Constructive proof of Baranyai's Theorem).

Suppose $n \geq 1$. Baranayai's Theorem guarantees that $\binom{[3n]}{3}$ has a decomposition into perfect matchings, without explicitly describing the matchings. The purpose of this exercise is to find a construction of the decomposition in the case $p := 3n - 1 \geq 3$ is a prime number.

(i). Consider the field $\mathbb{F}_p$ and denote by $\mathbb{F}_p^*$ the invertible elements, i.e. the set $\{1, 2, \ldots, p-1\}$. Define $\pi : \mathbb{F}_p^* \to \mathbb{F}_p$ by $\pi(x) = -\frac{1+x}{x}$. Show that π is injective and $\pi^3(x) = x$, for any $x \neq p - 1$.

(ii). Add an element u to $\mathbb{F}_p$ and extend π to $\{u, 0\}$ injectively in such a way that $\pi^3(x) = x$, for any $x \in \mathbb{F}_p \cup \{u\}$. Explain how this gives a perfect matching M_0 in $\binom{[3n]}{3}$. Using algebraic operations on M_0 find the remainining $\binom{3n-1}{2} - 1$ perfect matchings from Baranyai's theorem and prove that this is indeed a decomposition into disjoint perfect matchings.

Exercise 2.

Solve each of the following linear programs by making a diagram. In each case, specify whether the program is feasible and bounded, feasible and unbounded, or unfeasible. If the program is bounded, specify all optimal solutions. If it is unbounded, give an unbounded ray on which the objective function increases without limit.

1. Find the maximal value (if it exists) of $z = 3x + 4y$ subject to the following constraints

$$\begin{cases} x - y + 1 & \leq 0 \\ y & \leq 2 \\ -x & \leq 2 \end{cases}$$

2. Find the maximal value (if it exists) of $z = 2y - x$ subject to the following constraints

$$\begin{cases} 2y - x - 1 & \leq 0 \\ y - 2x & \leq 1 \\ -4y + 3x & \leq 0 \end{cases}$$

3. Find the maximal value (if it exists) of $z = x + y$ subject to the following constraints

$$\begin{cases} x - y + 1 & \leq 0 \\ 4x + 2y + 5 & \leq 0 \\ -x & \leq 0 \end{cases}$$

4. Find the maximal value (if it exists) of $z = x$ subject to the following constraints

$$\begin{cases} -x - y & \leq 0 \\ -x + y & \leq 0 \end{cases}$$

5. Find the maximal value (if it exists) of $z = x + y$ subject to the following constraint

$$x + y - 1 \leq 0$$

Exercise 3.

A building supply has two locations in town. The office receives orders from two customers, each requiring 3/4-inch plywood.

Customer A needs fifty sheets and Customer B needs seventy sheets.

The warehouse on the east side of town has eighty sheets in stock; the west-side warehouse has sixty sheets in stock. Delivery costs per sheet are as follows: 10 euro from the eastern warehouse to Customer A, 12 euro from the eastern warehouse to Customer B, 8 euro from the western warehouse to Customer A, and 11 euro from the western warehouse to Customer B.

Formulate the linear program associated to the problem of minimizing the cost of shipping.