Diskrete Mathematik II WS 2012 / 2013
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Exercise Sheet 8

Due date: Dec 11, 12:30 PM, beginning of exercises
NO LATE SUBMISSIONS!

You should try to solve and write up all the exercises. You are welcome to submit **at most** two neatly written exercises each week. You are encouraged to submit in pairs, but don't forget to mark the name of the scriber.

Exercise 1.

Let $A \subseteq \mathbb{R}^2$ be a set of points in the plane. An *annulus* (ring) is the region between two concentric circles. We define the *smallest enclosing annulus* of A as the annulus of smallest area that contains A. We allow points of A to lie on any of the two circles of the annulus, as well as the two circles to be identical. For example, if A lies on a circle, then the smallest enclosing annulus is given by two identical circles, and the area is 0.

Show that the problem of computing the smallest enclosing annulus of A can be solved by a linear program.

Exercise 2.

a) The n-dimensional crosspolytope is the set $C_n := \{x \in \mathbb{R}^n : |x_1|+|x_2|+\ldots+|x_n| \leq 1\}$. Thus $C_1 = [-1, 1]$ and C_2 is the parallelogram with vertices $(-1, 0), (1, 0), (0, 1)$ and $(0, -1)$. Express C_n as the solution set of a linear system of inequalities (meaning, a system of the form $Ax \leq b$).

b) Let $D = \{x \in \mathbb{R}^3 : |2x_1 - x_2 + 3x_3 + 1| + |x_2 + 2x_3 - 2| + |5x_1 - 3x_3| \leq 10$. Express D as the solution set of a linear system of inequalities.

Exercise 3.

We generalize the concept of independence number of a graph to hypergraphs as follows. Let $G = (V, E)$ be a hypergraph. We define $\alpha(G)$ as the maximum size of a subset $S \subseteq V(G)$, such that no hyperedge of G contains two vertices from S.

a) Formulate an integer linear program that finds the independence number of a hypergraph.

b) Formulate the LP relaxation of the integer program defined at a).

c) An automorphism of the hypergraph G is a bijection $\pi : V(G) \to V(G)$ such that $e \in E(G)$ if and only if $\pi(e) \in E(G)$, for any subset $e \subseteq V(G)$. The set of all automorphisms of G forms the automorphism group $Aut(G)$ of G. G is called *vertex-transitive* if for any $u, v \in V(G)$ there exists $\pi \in Aut(G)$ such that $\pi(u) = v$.

Let $\alpha_f(G)$ be the solution of the linear program defined at b). Prove that for a vertex-transitive hypergraph G we have $\alpha_f(G) = |V(G)|/t$, where t is the maximum size of a hyperedge in G.