Extremal Combinatorics Wi 2015–16
Tibor Szabó
Shagnik Das

# Exercise Sheet 1

**Due date: 12:30, Oct 21st, at the beginning of lecture.**
**Late submissions will meet a most unfortunate fate.**

You should try to solve all of the exercises below, but clearly mark which two solutions you would like us to grade – each problem is worth 10 points. We encourage you to submit in pairs, but please remember to indicate the author of each solution.

**Exercise 1** In the exercise you will prove the classic Erdős–Szekeres bound on the Ramsey numbers.

(i) Show that $R(s,2) = s$ and $R(2,t) = t$.

(ii) Show that for any $s,t \geq 3$, $R(s,t) \leq R(s-1,t) + R(s,t-1)$.

(iii) Prove, by induction, that $R(s,t) \leq \binom{s+t-2}{s-1}$ for any $s,t \geq 2$.

**Exercise 2** Improve our upper bound on the multicolour Ramsey numbers by showing

$$R_r(t_1,t_2,\ldots,t_r) \leq r^{\sum_{i=1}^r t_i}.$$

[Hint (to be read backwards): cimiM eht lanigiro ruoloc-owt foorp, daetsni fo gniyrt ot ecuder eht rebmun fo sruoloc.]

**Exercise 3** Deduce the finite Ramsey theorem from the infinite case. That is, using the fact that every red-/blue-colouring of $\binom{\mathbb{N}}{2}$ contains an infinite monochromatic clique, show that for every $t$ there is a finite $n$ such that every red-/blue-colouring of $E(K_n)$ contains a monochromatic $K_t$.

**Exercise 4** In this exercise we will provide lower bounds for the multicolour Ramsey problem.

(a) Show that if $r\binom{n}{t}r^{-\binom{t}{2}} < 1$, then $R_r(t,t,\ldots,t) \geq n+1$. Deduce the bound

$$R_r(t,t,\ldots,t) \geq te^{-1}r^{\frac{t-1}{2}-\frac{1}{t}}.$$

(b) Show that for any $n$, we have $R_r(t,t,\ldots,t) \geq n - r\binom{n}{t}r^{-\binom{t}{2}}$. What bound does this give for $R_r(t,t,\ldots,t)$?