Extremal Combinatorics Wi 2015–16
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Exercise Sheet 3

Due date: 12:30, Nov 4th, at the beginning of lecture.
Late submissions will be used to fuel a bonfire[1].

You should try to solve all of the exercises below, but clearly mark which two solutions you would like us to grade – each problem is worth 10 points. We encourage you to submit in pairs, but please remember to indicate the author of each solution.

Exercise 1 Show that if $n \geq R^{(3)}(t,t)$, then any set of n points in $\mathbb{R}^2$, no three collinear, contains a subset of t points in convex position.

[Hint (to be read backwards): yrT gniruoloc selpirt desab no tahw redro eht stniop raeppa ni nehw gniog dnuora eht elgnairt ni a esiwkcolc noitcerid.]

Exercise 2 Use Ramsey's theorem to show that for any infinite sequence $(x_1, x_2, x_3, \ldots)$ of real numbers, there is an infinite monotone[2] subsequence $(x_{i_1}, x_{i_2}, x_{i_3}, \ldots)$ given by some indices $i_1 < i_2 < i_3 < \ldots$.

If we instead have a finite sequence $(x_1, x_2, x_3, \ldots, x_n)$, how large a monotone subsequence are we guaranteed to find?

Exercise 3

(i) Show that whenever the natural numbers $\mathbb{N}$ are finitely coloured, one can find monochromatic $x, y, z \in \mathbb{N}$ such that $x + y = z$.

[Hint (to be read backwards): enifeD na etairporppa gniruoloc fo *sriap* fo larutan srebmun.]

(ii) Show that for every $r \geq 1$ there is some $n = n(r)$ such that if the sets in $2^{[n]}$ (that is, all subsets of the n-element set $[n]$) are coloured with r colours, one can find non-empty disjoint sets X and Y such that X, Y, and $X \cup Y$ all receive the same colour.

Exercise 4 Let H be a k-graph, for some $k \geq 2$, with the property that $|e \cap f| \neq 1$ for any two edges $e, f \in E(H)$. Show that H is two-colourable.

[1]Remember, remember!
The fifth of November,
The Gunpowder treason and plot;
I know of no reason
Why the Gunpowder treason
Should ever be forgot!

[2]A sequence is *monotone* if it is either non-increasing or non-decreasing.