Freie Universität Berlin
Institut für Mathematik
Extremal Combinatorics

Winter Semester 2015-16
Dec 16th, 2015

Lecturer: Shagnik Das

Tutor: Tuan Tran

Exercise	1.	2.	3.	4.	5.
Points					

PRACTICE EXAM

You may use this practice exam to test your knowledge of the material covered in the course so far. It will not form part of your grade in any way, but if you would like to receive feedback on your solutions, please submit them to the tutor box of Shagnik Das on the first floor of Arnimallee 3 by the end of Monday, January 4th, 2016. We would recommend you take this under exam conditions — three hours, no notes.

Show all your work and state precisely any theorems you use from the lectures.

Exercise 1 [10 points]

(a) Define the Ramsey number $R^{(3)}(t,t)$, and show that it is finite.

(b) Use a random colouring to show $R^{(3)}(t,t) \geq (1-o(1))\,\frac{t}{e}2^{(t-1)(t-2)/6}$.

Exercise 2 [10 points]

Given an arbitrary graph F, define the Turán number $\mathrm{ex}(n,F)$. Show that when $n \geq 4$ and F is the graph depicted in Figure 1, we have $\mathrm{ex}(n,F) = \left\lfloor \frac{n^2}{4} \right\rfloor$.

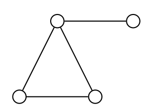

Figure 1: The graph F.

Exercise 3 [10 points]

(a) State the Szemerédi Regularity Lemma, defining all necessary terms.

(b) State and prove the Slicing Lemma.

Exercise 4 [10 points]

Show that for every $\varepsilon > 0$, there is some $n_0 = n_0(\varepsilon)$ such that if $n \geq n_0$ and $A \subset [n]^2$ has size $|A| \geq \varepsilon n^2$, then A contains three points $\{(x,y),(x+h,y),(x,y+2h)\}$, where $x, y \in [n]$ and $h \neq 0$.

Exercise 5 [10 points]

There is a town containing n people, where n is an odd number. The people of this town form clubs, with every club having an even number of people, and any two different clubs having an odd number of people in common.

(i) Show that it is possible to have n clubs.

(ii) Prove there cannot be more than n clubs. You should prove any theorems from lecture that you use.