Freie Universität Berlin
Institut für Mathematik
Extremal Combinatorics

Winter Semester 2017-18
Dec 18th, 2017

Lecturer: Tibor Szabó

Tutor: Anurag Bishnoi

Exercise	1.	2.	3.	4.	5.	6.
Points						

PRACTICE EXAM

You may use this practice exam to test your knowledge of the material covered in the course so far. It will not form part of your grade in any way, but if you would like to receive feedback on your solutions, please submit them by January 9th, 2018. We will discuss the solutions of these questions in the tutorials on January 9th and 10th. We would recommend you take this under exam conditions — three hours, no notes.

Show all your work and state precisely any theorems you use from the lectures (unless the exercise itself is a theorem from the lecture.)

Exercise 1 [10 points]

(1) State the definition of the multicolor Ramsey number $R_r(t_1, t_2, \ldots, t_r)$.

(2) Prove that $R_r(t_1, t_2, \ldots, t_r) \leq r^{1+\sum_{i=1}^{r}(t_i-2)}$.

Exercise 2 [10 points]
Let $m_B(k) = \min\{|\mathcal{H}| : \mathcal{H}$ is a k-uniform hypergraph which is not two-colorable$\}$, where $|\mathcal{H}|$ denotes the number of hyperedges in $\mathcal{H}$.

(1) Prove that $m_B(2) = 3$.

(2) Prove that $m_B(3) = 7$.

Exercise 3 [10 points]

(1) State the definition of ε-regularity.

(2) Let G be a graph and let $A, B \subseteq V(G)$ be two disjoint sets of vertices. Let $\varepsilon > 0$ and say A, B are ε-regular with the density $d(A, B)$ at least d, and $d > \epsilon$. Prove that for any $Y \subseteq A$ with $|Y| \geq \varepsilon|A|$, we have

$$|\{x \in B : |N(x, Y)| \geq (d - \varepsilon)|Y|\}| \geq (1 - \varepsilon)|B|,$$

where $N(x, Y)$ denotes the set of vertices in Y which are adjacent to x.

Exercise 4 [10 points]

(1) Let H be any graph and n a positive integer. State the definition of the Turán number $\mathrm{ex}(n, H)$.

(2) Prove that $\mathrm{ex}(n, C_4) \leq O(n^{3/2})$.

Exercise 5 [10 points]

There is a town containing n people and the people of this town form clubs, with every club having an even number of people, and any two different clubs having an odd number of people in common.

(1) Prove that there cannot be more than $n+1$ clubs and give a construction of $n-1$ such clubs.

(2) When n is odd prove that there cannot be more than n clubs, and give a construction of n such clubs.

Exercise 6 [10 points]

In an acoustic wave imaging experiment, Tajel uses a 4.5V amplifier that requires 3 batteries, of 1.5V each, to work. She sees that there are 8 batteries in her office drawer, but she remembers that only 4 of them work. There is no way of telling them apart without testing them in the amplifier (the amplifier works only when it has three working batteries in it). Prove that Tajel can find a working triple of batteries in 20 trials.