Algorithmic Combinatorics — Wi 2018–19

Tibor Szabó
Anurag Bishnoi

Exercise Sheet 0

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The following exercises are designed to refresh your knowledge of some topics from Discrete Mathematics I. You are encouraged to try to solve all of these problems, and should feel free to work on them with a partner or in a small group.

Exercise 1 For each pair of integer valued functions below determine whether $f = o(g)$, $g = O(f)$, $f = O(g)$, $f = \Omega(g)$, $f = \Theta(g)$ or $f \sim g$.

(a) $f(n) = 2^n$ and $g(n) = n!$.

(b) $f(n) = \sum_{i=1}^{n} 1/i$, and $g(n) = \log_2 n$.

(c) $f(n) = n^{1/\log n}$, and $g(n) = \log_2 n$.

Exercise 2 Let S be a finite set of points in $\mathbb{R}^2$. Let k be the largest possible size of a subset A of S for which $|A| = |\{x : (x,y) \in A\}| = |\{y : (x,y) \in A\}|$. Prove that you can find k horizontal and vertical lines that cover all points of S.

(Bonus) Prove that in the n-dimensional generalisation of the problem, you can find $(n-1)k$ hyperplanes parallel to the axes that cover all the points.

Exercise 3 Show that for any set of S of n integers you can find a nonempty subset T of S with the property that the sum of all integers in T is divisible by n.

Exercise 4 Find the largest number of edges in n-vertex graph that has no subgraph isomorphic to $K_{1,3}$.

Exercise 5 Given a graph G, the diameter $\mathrm{diam}(G)$ is defined as $\max\{d(x,y) : x, y \in V(G)\}$, where $d(\cdot,\cdot)$ is the distance function of the graph, and the girth $\gamma(G)$ is defined as the length of the shortest cycle in G. Prove that for any simple graph G,

$$\gamma(G) \le 2 \cdot \mathrm{diam}(G) + 1.$$

If G is bipartite, then show that $\gamma(G) \le 2 \cdot \mathrm{diam}(G)$.

Exercise 6 A graph is called self-complimentary if it is isomorphic to its complement. For example, the cycle C_5 is self complementary. Prove that a self-complementary graph with n vertices exists if and only if $n \equiv 0 \pmod 4$ or $n \equiv 1 \pmod 4$.