Algorithmic Combinatorics Wi 2018–19
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Exercise Sheet 8

Due date: 14:15, 11th December

You should try to solve all of the exercises below, but clearly mark which two solutions you would like us to grade – each problem is worth 10 points. We encourage you to submit in pairs, but please remember to indicate the author of each solution. Until submission, you are forbidden to look at the solutions of any of these exercises on the internet.

Exercise 1 Given a set $\mathcal{L}$ of lines in the plane $\mathbb{R}^2$, such that no three lines are through a common point, construct a graph G as follows: the vertex set is the set of intersection points of the lines in $\mathcal{L}$ and two points are adjacent if they are consecutive points on a common line. Prove that $\chi(G) \leq 3$.

Exercise 2 Prove that for every cycle the list chromatic number and the chromatic number are equal.[1]

Exercise 3 Show that every triangle-free planar graph has list chromatic number at most 4.

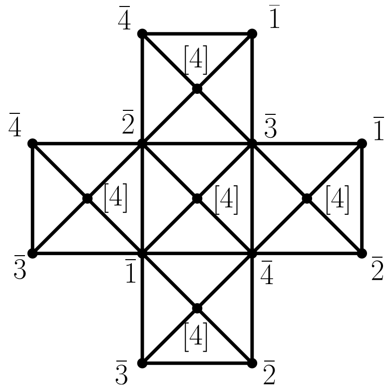
(Bonus) Construct a triangle-free planar graph G with $\chi_l(G) = 4$.

Exercise 4 A graph is called *outerplanar* if it has a planar drawing in which all the vertices are contained in the outer face of the drawing. Prove that the list chromatic number of every outerplanar graph is at most 3.

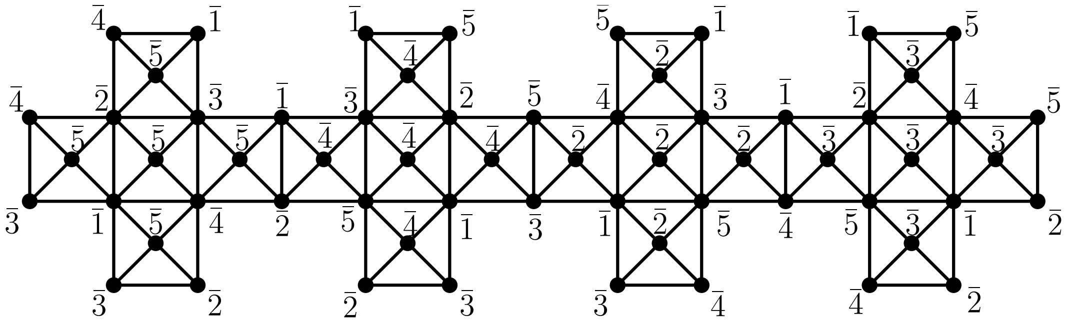
Exercise 5 In this exercise we will prove that Thomassen's theorem is tight; that is, there are planar graphs that are not 4-choosable.

(a) Show that the following graph has no proper coloring from the assigned lists. (In this part of the exercise, $\bar{i}$ denotes the set $[4] \setminus \{i\}$.)

[1] In particular, this shows that $\chi_l(K_{2,2,}) = 2$.



(b) Let G be the graph obtained from the graph below by adding an extra vertex to the outside face and connecting it to all the vertices on the boundary. Show that if the new vertex is assigned the list $\bar{1}$, then there is no proper list coloring of G from the assigned lists.[2] (In this part of the exercise, $\bar{i}$ denotes the set $[5] \setminus \{i\}$.)



(c) Determine the chromatic number of this graph.

[2]This graph appeared in a paper of Maryam Mirzakhani in 1996, just as she began her undergraduate studies.