Freie Universität Berlin
Institut für Mathematik
Algorithmic Combinatorics

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Exercise	1.	2.	3.	4.	5.	6.
Points						

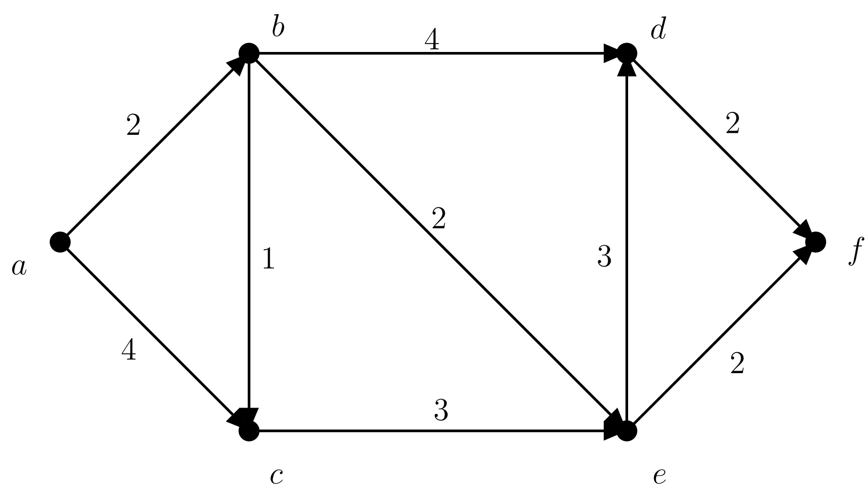
PRACTICE EXAM

Show all your work and state precisely the theorems you are using from the lecture. Ideally, try to solve the sheet within a time limit of 180 minutes, without using any books, notes, etc ... (but of course this is not mandatory if you feel it would not yet make sense this way). It will be graded like the Final Exam, but the points do not count towards your exercise credit.

Exercise 1 [10 points]

(1) Describe precisely Dijkstra's algorithm.

(2) Using Dijkstra's algorithm find the length of the shortest paths from the vertex a to all other vertices in the following edge weighted graph. State all the steps clearly.



Exercise 2 [10 points]

(1) Describe the following classes of decision problems, (i) NP, (ii) NP-Complete.

(2) A k-CNF formula is the 'and' of a number of clauses with each clause being an 'or' of exactly k literals, where a literal is either a variable x_i or its negation $\neg x_i$. For example, $f(x_1, \ldots, x_5) = (x_1 \vee x_2 \vee \neg x_3) \wedge (x_2 \vee \neg x_3 \vee x_5) \wedge (x_1 \vee \neg x_4 \vee \neg x_5)$ is

a 3-CNF with three clauses. We say that a k-CNF formula is satisfiable if there exists a True/False assignment to the variables that makes the formula True. The k-SAT problem is the decision problem about whether a given a k-CNF is satisfiable or not.

Assuming 3-SAT is NP-complete, prove that k-SAT is NP-complete for all $k \geq 3$.

Exercise 3 [10 points]

(1) Prove that every 3-regular graph G with edge connectivity number $\kappa'(G) > 1$ has a perfect matching.

(2) Give an example of a 3-regular graph that has no perfect matching.

Exercise 4 [10 points]

(1) Define the list chromatic number $\chi_\ell(G)$ of a graph G.

(2) Prove that for every planar graph G we have $\chi_\ell(G) \leq 5$.

Exercise 5 [10 points]

(1) Define the chromatic index $\chi'(G)$ of a multigraph G, and state Vizing's theorem.

(2) Let G be a d-regular connected graph that has a cut vertex. Determine $\chi'(G)$.

Exercise 6 [10 points]

(1) State the stable matching problem and describe the Gale-Shapley proposal algorithm for finding a stable matching.

(2) Let S be the set of stable matchings between n men and n women, where each person has a ranking of all n members of the other gender. A woman w is a valid partner of a man m if there exists a matching in S in which m and w are paired together. A matching is called *man optimal* if if each man receives his best valid partner. Prove that the algorithm in which the men propose to women is man optimal.