Freie Universität Berlin
Institut für Mathematik
Extremal Combinatorics

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Winter Semester 2019-20

Exercise	1.	2.	3.	4.	5.	6.
Points						

PRACTICE EXAM

You may use this practice exam to test your knowledge of the material covered in the course so far. It will not form part of your grade in any way, but if you would like to receive feedback on your solutions, please submit them for grading. We recommend that you submit them by the 7th of January 2020 as it will be discussed during the exercises classes on Janury 7th and on January 9th. In addition we will release a new exercise sheet on January 8th. However we expect to grade the practice exams during the second week of classes, hence any exam submitted by 09:00 on January 13th will be graded. We would recommend that you take this exam under exam conditions three hours, no notes.

Instructions Solve all the questions. Each question is worth 10 points. Show all your work and state precisely the theorems you are using from the lecture. No notes are allowed. The time limit is 3 hours.

Notation: $\mathbb{N} = \{1, 2, 3, ...\}$.

Question 1 [10 points]

(a) For $t_1, t_2, t_3 \in \mathbb{N}$ define the Ramsey number $R(t_1, t_2, t_3)$.

(b) For $k \in \mathbb{N}$ prove that

$$R(k,k,k) \geq (1 - o_k(1)) \frac{k}{3e} \sqrt{3}^k.$$

Question 2 [10 points]

(a) Let H be a fixed given graph H and $n \in \mathbb{N}$. Define $ex(n, H)$.

(b) Let $1 \leq s \leq t$. Prove that $ex(n, K_{s,t}) \leq c_{s,t} n^{2-1/s}$ for some explicit constant $c_{s,t}$ that may depend on s, t.

Question 3 [10 points]

(a) State the definition of ϵ-regularity.

(b) Let $A, B, C \subseteq V(G)$ be pairwise disjoint sets in some graph G such that the pairs $\{A, B\}, \{B, C\}, \{A, C\}$ are all ϵ-regular with densities $d(A, B), d(A, C), d(C, B) \geq 2\epsilon$. Show that the number of triangles in G is at least

$$(1 - 2\epsilon)(d(A, B) - \epsilon)(d(A, C) - \epsilon)(d(B, C) - \epsilon)|A| \cdot |B| \cdot |C|.$$

Question 4 [10 points]

Show that whenever $\mathbb{N}$ is 2-colored there exists a monochromatic solution to the equation

$$x + 2.5y = z.$$

Question 5 [10 points]

Let $\mathcal{F}$ be a family of subsets of $[n]$ such that,

(a) $|F| \neq 0 \mod 6$, for $F \in \mathcal{F}$.

(b) $|F_i \cap F_j| = 0 \mod 6$, for every pair of distinct $F_i, F_j \in \mathcal{F}$.

Show that $|\mathcal{F}| \leq 2n$.

Question 6 [10 points]

A set S of points in $\mathbb{R}^n$ is called a **two-distance set** if the distance between pairwise distinct points of S takes only two distinct values. Let $m(n)$ be the maximum size of a two-distance set in $\mathbb{R}^n$.

(a) Prove that $m(n) \geq n(n-1)/2$.

(b) Prove that there exists a constant $C > 0$ such that $m(n) \leq O(n^C)$.

(c) (Not part of the exam) Show that C in part (b) can be taken to be equal to 2.