Extremal Combinatorics Wi 2019–20
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Bonus Exercise Sheet[1]

Due date: 14:15, 4th February[2]

You should try to solve all of the exercises below, but clearly mark which two solutions you would like us to grade – each problem is worth 10 points. We encourage you to submit in pairs, but please remember to indicate the author of each solution.

Exercise 1 Let H be a 3-uniform hypergraph with bi-partition $V(H) = A \cup B$ (no edge of G is contained in either A or B). For $A' \subseteq A$ let

$$N(A') = \{\{x, y\} \in B \times B : \{a, x, y\} \in E(H) \text{ for some } a \in A'\}.$$

Show that if $|N(A')|$ spans a matching of size $2(|A'|-1)+1$ (in the corresponding 2-uniform hypergraph) for every $A' \subseteq A$ then H contains a matching that saturates A.

Exercise 2 Let G be an $(2k)$-partite graph, with each part having n vertices, of maximum degree Δ.

(i) Show that if $n > 2\Delta - \frac{\Delta}{k}$, then G must have an independent transversal.

(ii) Show that the construction given in the lecture shows that this is indeed optimal, that is there exist a $(2k)$-partite graph with parts of size $2\Delta - \lceil \frac{\Delta}{k} \rceil$ and maximum degree Δ that has no independent transversals.

[1]For the Aktive Teilnahmecredit for this course, you are required to obtain at least 60% of the total available points (not counting this one). This means you are required to obtain 144 points in total. If you have not then you may submit this hw sheet for grading. Any points gained will be added to your current ones and count towards the 144 points that you need. Of course any solutions submitted will be graded so you may submit solutions even if you have fullfill this requirement.

[2]Please submit the exercise sheet before 14:15 on Tuesday. You can submit it in the tutor box of Michael Anastos (box number B8, in front of lecture hall 001, Arnimallee 3-5) or at the beginning of the exercise class on Tuesday or electronically at manastos@zedat.fu-berlin.de

Exercise 3 After almost 15 years of operation of the Berlin Mathematical School (BMS), the Head of BMS (let's call him Mr X), decided to relocate its headquarters. Upon hearing so each of the mathematical departments of FU, HU and TU submitted a map to Mr X with their prefered areas marked in red (let's denote those areas by R_{FU}, R_{HU} and R_{TU} respectively). Upon examining the 3 maps Mr X realised that for every $S \subseteq \{FU, HU, TU\}$ the set of points P_S that lie in the convex hull spanned by the buildings of the corresponding departments are marked red by at least 1 of the universities in S. In addition all of R_{FU}, R_{HU} and R_{TU} are closed sets. Can Mr X choose a new location to relocate the headquarters of BMS that will satisfy all 3 departments?

Exercise 4 Deduce the 2-dimensional case of Sperner's Lemma from the following Statement:

- Let $T \subset \mathbb{R}^2$ be a triangle and $f : T \mapsto T$ be a continuous function. Then there exists a point $x \in T$ such that $f(x) = x$.