

Exercise Sheet 3

Due date: 14:15, 5th November¹

You should try to solve all of the exercises below, but clearly mark which two solutions you would like us to grade – each problem is worth 10 points. We encourage you to submit in pairs, but please remember to indicate the author of each solution.

Exercise 1 Let H be a k -uniform hypergraph, for some $k \geq 2$, with the property that $|e \cap f| \neq 1$ for any two edges $e, f \in E(H)$. Show that H is two-colourable.

Exercise 2

- (i) Show that whenever the natural numbers \mathbb{N} are finitely coloured, one can find monochromatic $x, y, z \in \mathbb{N}$ such that $x + y = z$.
- (ii) Show that whenever the natural numbers \mathbb{N} are finitely coloured, one can find **distinct** monochromatic $x, y, z \in \mathbb{N}$ such that $x + y = z$.

Exercise 3 Show that for $t \geq 3$, $HE(t) \leq R^{(3)}(t, t)$.

Exercise 4 Given a colouring $c : \binom{\mathbb{N}}{3} \mapsto C$ a set $S \subset \mathbb{N}$ is said to be

- (i) rainbow if no two triples have the same color.
- (ii) c -left-injective if there is a map $c^* : \mathbb{N} \mapsto C$ such that $c(i, j, k) = c^*(i)$ for every $i, j, k \in S$ with $i < j < k$.
- (iii) c -middle-injective if there is a map $c^* : \mathbb{N} \mapsto C$ such that $c(i, j, k) = c^*(j)$ for every $i, j, k \in S$ with $i < j < k$.
- (iv) c -right-injective if there is a map $c^* : \mathbb{N} \mapsto C$ such that $c(i, j, k) = c^*(k)$ for every $i, j, k \in S$ with $i < j < k$.

Prove or disprove the following statement. Let $c : \binom{\mathbb{N}}{3} \mapsto C$ be a coloring. Then there exists some infinite subset $S \subset \mathbb{N}$ such that

¹Please submit the exercise sheet before 14:15 on Tuesday. You can submit it in the tutor box of Michael Anastos (box number B8, in front of lecture hall 001, Arnimallee 3-5) or at the beginning of the exercise class on Tuesday or electronically at manastos@zedat.fu-berlin.de

- (i) S is monochromatic, or
- (ii) S is rainbow, or
- (iii) S is c -left-injective, or
- (iv) S is c -middle-injective, or
- (v) S is c -right-injective.

Hint to Exercise 2(i): the hint starts here define an appropriate coloring of pairs of natural numbers