Exercise Sheet 7.

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Discrete Mathematics III — Constructive Combinatorics, Summer 2012
Due date: June 27th (Wednesday) by 8:30, at the beginning of the exercise class.

Problem 1. Show from scratch that $R(4,4) = 18$.

Problem 2. Prove that the Abbott product is an explicit construction in the "efficient", computer scientific sense. That is, show that for any n you are able to construct the adjacency matrix of a $\sqrt[10]{n}$-Ramsey graph G_n on n vertices, in time polynomial in n. Give a concrete upper bound, bounded by a polynomial in n, on the number of steps this takes.
Even more, show that given any two vertices i and j from the vertex set $[n]$ of G_n, you can tell whether they are adjacent in time polynomial in $\log n$. This question is motivated by the fact that describing i and j only takes $\log n$ bits.

Problem 3. Show that d-wise independence of a sample space implies its d'-independence for every $d' \leq d$.

Problem 4. Carry out the calculation about how many vertices does the k-Ramsey graph constructed via d-wise independent sample spaces has in terms of k.