Structural Graph Theory Autumn 2016-17
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Exercise Sheet 1

Due date: 08:00am, October 26th.

You should try to solve all of the exercises below. In order to get a 'plus' for this sheet you should submit correct solutions for exercises worth at least half of the total (12) number of points. You can submit solutions in print (my office) or by email. The solutions will be graded.

Exercise 1 [`2 points`]

(i) Prove that if $H \prec_{\text{top}} G$, then $H \prec G$.
(ii) Let H and G be a graph depicted at the figure. Prove that H is not a topological minor of G.
(iii) Prove that if $\Delta(H) \leqslant 3$ and $H \prec G$, then $H \prec_{\text{top}} G$.

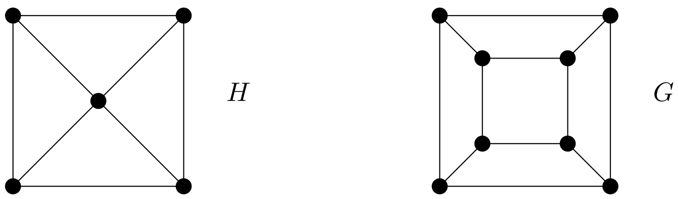

Figure 1: Why H is not a topological minor of G?

Exercise 2 [`1 point`] Given Euler's Formula for plane graphs show that for every bipartite plane and connected graph G we have

$$|E(G)| \leqslant 2n - 4.$$

Exercise 3 [`1 point`] Prove that if G is a 3-connected plane graph, then for every $v \in V(G)$ when we remove v from the drawing, then the new face in the resulting drawing of $G - v$ is bounded by a cycle.

Exercise 4 [`3 points`] Given Kuratowski's Theorem for 3-connected plane graphs, that is in particular for every 3-connected graph G with $K_{3,3} \not\prec G$ and $K_5 \not\prec G$, we have that G is planar. Show the statement also for graphs G that are not 3-connected.

Exercise 5 [`2 points`] A football is made of pentagons and hexagons, not necessarily of regular shape. They are sewn together so that their seams form a cubic graph. How many pentagons does the football have?

Exercise 6 [`3 points`] Let $\mathcal{T}$ be a set of subtrees of a tree T, and $k \in \mathbb{N}$.

(i) Show that if the trees in $\mathcal{T}$ have pairwise non-empty intersection then their overall intersection $\bigcap \mathcal{T}$ is non-empty.

(ii) Show that either $\mathcal{T}$ contains k disjoint trees or there is a set of at most $k-1$ vertices of T meeting every tree in $\mathcal{T}$.