Structural Graph Theory Winter 2016-17
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Exercise Sheet 2

Due date: 08:00am, November 8th.

You should try to solve all of the exercises below. In order to get a 'plus' for this sheet you should submit correct solutions for exercises worth at least half of the total (8) number of points. You can submit solutions in print (my office) or by email. The solutions will be graded.

Exercise 1 [`2 points`] Prove that every graph of average degree at least 2^{r-2} has a K_r-minor. Hint: Try to optimize the proof of Mader's Theorem for topological minors that was lectured on October 18th.

Exercise 2 [`1 points`] Let $k \geqslant 2$. Show that every k-connected graph on at least $2k$ vertices contains a cycle of length at least $2k$.

Exercise 3 [`1 points`] Let $k \geqslant 2$. Show that in a k-connected graph any k vertices lie on a common cycle.

Exercise 4 [`1 points`] Let X an X' be minimal separators in G such that X meets at least two components of $G - X'$. Show that X' meets all the components of $G - X$, and that X meets all the components of $G - X'$.

Exercise 4 [`1 points`] Show that k-linked graphs are $(2k-1)$-connected. Are they even $2k$-connected?

Exercise 5 [`2 points`] A graph is *chordal* if it does not have an induced cycle of length $\geqslant 4$. Show that the following statements are equivalent:

(i) G is chordal;

(ii) there is a tree T and a family $\mathcal{T} = \{T_v\}_{v \in V(G)}$ of subtrees of T such that $uv \in E(G)$ if and only, if $T_u \cap T_v$ is non-empty.