Structual Graph Theory Winter 2016-17
Piotr Micek

Exercise Sheet 3

Due date: 08:00am, November 16

You should try to solve all of the exercises below. In order to get a 'plus' for this sheet you should submit correct solutions for exercises worth at least 'half' of the total number of points, which is say 3 for this week. You can submit solutions in print (my office) or by email. The solutions will be graded.

Exercise 1 [`2 points`] Hajós Conjecture, which is known to be wrong states that: for every $r \geqslant 1$ and every graph G, if $r \leqslant \chi(G)$ then $K_r \prec_{\text{top}} G$. Construct a counterexample for $r = 8$ (see a figure for inspiration) and later for $r = 7$. It is still an open problem, if the conjecture is true for $r = 5, 6$

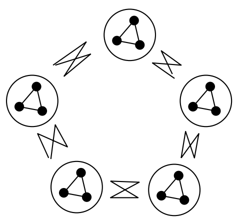

Figure 1: Hint about an counterexample contruction for $r = 8$. Proposed by Catlin in 1978.

Exercise 2 [`1 points`] Every graph G has a subgraph of minimum degree at least $\chi(G) - 1$.

Exercise 3 [`2 points`] It is known that there is $h : \mathbb{N} \to \mathbb{N}$ such that for every graph G with $d(G) \geqslant h(r)$, we have $K_r \prec_{\text{top}} G$. Show that any such h must satisfy $h(r) > \frac{1}{8}r^2$ for all even r.

Exercise 4 [`2 points`] A graph with at least three vertices is edge-maximal without a K_4 minor if and only if it can be constructed recursively from triangles by pasting along edges. (If G is a graph with induced subgraphs G_1, G_2 and S, such that $G = G_1 \cup G_2$ and $S = G_1 \cap G_2$, we say that G arises from G_1 and G_2 by *pasting* these graphs together *along* S.) This should imply Hadwiger conjecture for $r = 4$. Why?