Structural Graph Theory | Winter 2016-17
Piotr Micek

Exercise Sheet 5

Due date: 08:00am, December 7th.

You should try to solve all of the exercises below. In order to get a 'plus' for this sheet you should submit correct solutions for exercises worth at least half of the total (4) number of points. You can submit solutions in print (my office) or by email. The solutions will be graded.

Exercise 1 [`2 points`] For a positive integer k, a set S of vertices in a graph G is *k-nice* if for every set $X \subseteq V(G)$ such that $|X| < k$ there is a component of $G - X$ that contains more than half of the vertices in S. The *niceness* of G, denoted by $\mathrm{nice}(G)$, is the maximum integer k for which G contains a k-nice set. Prove that for every graph G

$$\mathrm{nice}(G) \leqslant \mathrm{bramble}(G) \leqslant 2\,\mathrm{nice}(G).$$

Are these inequlities tight?

Exercise 2 [`1 points`] Can the tree-width of a subdivision of a graph G be smaller than $\mathrm{tw}(G)$? Can it be larger?

Exercise 3 [`2 points`] The *circumference* of a graph is the length of any longest cycle in a graph. Show that if a graph has circumference $k \neq 0$, then its tree-width is at most $k - 1$.

Exercise 4 [`1 points`] A graph is *outerpalnar* if it has a planar drawing in which every vertex lies on the outerface. Show that outerplnar graphs can have arbitrarily large tree-width, or find the best upper bound.

Exercise 5 [`2 points`] Device an algorithm that given a graph G and (a smooth) tree decomposition of G of width w, determines whether G is 3-colorable in time $|V(G)| \cdot O(3^w \cdot \mathrm{polynomial}(w))$.