Structural Graph Theory | Winter 2016-17
Piotr Micek

Exercise Sheet 8

Due date: 08:00am, January 25.

You should try to solve all of the exercises below. In order to get a 'plus' for this sheet you should submit correct solutions for exercises worth at least half (meaning 2) of the total (3) number of points. You can submit solutions in print (my office) or by email. The solutions will be graded.

Exercise 1 [`1 points`] Let $\mathcal{K}_G(X,Y)$ be the maximum number of disjoint (X,Y)-paths in a graph G. A sequence $(H_1,\ldots,H_t)$ of subgraphs of G is a *k-dissection* of G if $(H_1 \cup \ldots H_i, H_{i+1} \cup \ldots \cup H_t)$ is a k-separation for each $i \in \{1,\ldots,t-1\}$.

Let G be a graph and $X, Y \subseteq V(G)$. Let $P_1,\ldots,P_k$ be disjoint (X,Y)-paths in G and let $P_1 = (v_0,\ldots,v_t)$ where $v_0 \in X$ and $v_t \in Y$. Show that: If $\mathcal{K}_{G-v_i}(X,Y) < k$ for each $i \in \{1,\ldots,t-1\}$, then there is a k-dissection $(H_1,\ldots,H_t)$ such that

(i) $X \subseteq V(H_i)$, $Y \subseteq V(H_t)$, and
(ii) $v_i \in V(H_i) \cap V(H_{i+1})$ for each $i \in \{1,\ldots,t-1\}$.

Exercise 2 [`1 points`] Let $t,n > 0$ and let $k = k(t,n)$ be sufficiently large. Let $S_1,\ldots,S_t$ and $T_1,\ldots,T_t$ be sets of vertices in a graph G such that $\mathcal{K}_G(S_i,T_i) \geqslant k$ for each $i \in \{1,\ldots,t\}$. Show that either

(i) there exist disjoint paths $P_1,\ldots,P_t$ where P_i is an (S_i,T_i)-path, or
(ii) $n \times n \prec G$.

Hint: Come for a lecture to see the Key Lemma and then iteratively apply it here.

Exercise 3 [`1 points`] Let H be a fixed graph. We say that $X \subseteq V(G)$ *covers* models of H, if $G - X$ has no copy of H as a minor. A graph H has the *Erdős-Pósa minors-property* if for every graph G the number of vertices in G needed to cover models of H is bounded by a function of the maximum number of disjoint models of H in G.

Show that: If H is planar, then H has the Erdős-Pósa minors-property.