Discrete Mathematics III So 2016
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Exercise Sheet 1

Due date: 12:30, Apr 27th, at the beginning of the exercise class.

You should try to solve all of the exercises below, and submit two solutions to be graded — each problem is worth 10 points. We encourage you to submit in pairs, but please remember to indicate the author of each individual solution.

Exercise 1 Suppose $n \geq 4$ and let H be an n-uniform hypergraph with at most $\frac{4^{n-1}}{3^n}$ edges. Prove that there is a coloring of the vertices of H by 4 colors so that in every edge all four colors are represented.

Exercise 2 From the lecture we know that for every integer $k > 0$ there is a tournament $T_k = (V, E)$ with $|V| > k$ such that for every set U of at most k vertices of T_k there is a vertex v so that all directed arcs $\{(v, u) \ : \ u \in U\}$ are in E. Show that each such tournament contains at least $\Omega(k2^k)$ vertices.
Hint: Probability theory is not necessarily needed.

Exercise 3 Let F be a finite collection of binary strings of finite lengths and assume no member of F is a prefix of another one. Let N_i denote the number of strings of length i in F. Prove that

$$\sum_i \frac{N_i}{2^i} \leq 1.$$

Exercise 4 Prove that there is an absolute constant $c > 0$ with the following property. Let A be an n by n matrix with pairwise distinct entries. Then there is a permutation of the rows of A so that no column in the permuted matrix contains an increasing subsequence of length at least $c\sqrt{n}$.

Exercise 5 Prove that there is a positive constant c so that every set A of n real nonzero reals contains a subset $B \subset A$ of size $|B| \geq cn$ so that there are no $b_1, b_2, b_3, b_4 \in B$ satisfying

$$b_1 + 2b_2 = 2b_3 + 2b_4.$$