Discrete Mathematics III So 2016
Tibor Szabó

Exercise Sheet 2

Due date: 12:30, May 4th, at the beginning of the exercise class.

You should try to solve all of the exercises below, and submit two solutions to be graded — each problem is worth 10 points. We encourage you to submit in pairs, but please remember to indicate the author of each individual solution.

Exercise 1 As seen in the lecture, the Ramsey number $R(k,k)$ satisfies

$$R(k,k) > n - \binom{n}{k} 2^{1-\binom{k}{2}}$$

for every integer n. Conclude that

$$R(k,k) \geq (1-o(1))\frac{k}{e}2^{\frac{k}{2}}.$$

Exercise 2 Prove that every three-uniform hypergraph with n vertices and $m \geq \frac{n}{3}$ edges contains an independent set (i.e. a set of vertices containing no edges) of size at least

$$\frac{2n^{\frac{3}{2}}}{3\sqrt{3}\sqrt{m}}.$$

Exercise 3 Let H be a graph, and let $n > |V(H)|$ be an integer. Suppose there is a graph on n vertices and t edges containing no copy of H, and suppose that $tk > n^2 \log_e n$ Show that there is a coloring of the edges of the complete graph on n vertices by k colors with no monochromatic copy of H.

Exercise 4 Let F be a finite collection collection of binary strings of finite length and assume that no two distinct concatenations of two finite sequences of codewords result in the same binary sequence. Let N_i denote the number of strings of length i in F. Prove that

$$\sum_i \frac{N_i}{2^i} \leq 1.$$