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A simple proof of the representation of bipartite planar graphs as the contact graphs of orthogonal straight line segments ☆

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Let S be a collection of closed, orthogonal (i.e., vertical and horizontal) straight line segments. The contact graph of a collection S of segments with pairwise disjoint interiors has S as its set of vertices and two segments are adjacent if and only if they touch. De Freysseix, de Mendez, and Pach [1] gave a linear algorithm for representing a bipartite planar graph as the contact graph of a set of orthogonal straight line segments. Their method is based on a linear algorithm for constructing bipolar orientations of 2-connected planar graphs.

In this note we give a direct and simple linear time algorithm for representing a bipartite planar graph as the contact graph of a set of orthogonal, closed straight line segments.

Theorem 1. *There is linear time algorithm for representing bipartite planar graphs as the contact graphs of orthogonal, closed straight line segments.*

Proof. We show by induction on the number of vertices of the graph G that the required representation exists. In fact we show that there exists a representation in which the segments representing the vertices of the outer face form a "ladder" in the segment representation of the graph.

To start, let x, y be two adjacent vertices in the outer face; represent x, y by horizontal, vertical segments on the x-, y-axis, respectively (see Fig. 1). Take any node u in the outer face of the graph. Let this vertex be adjacent to d vertices, say $1, 2, \ldots, d$. Delete u and all the links adjacent to it. The resulting graph, call it H, is planar bipartite. By the induction hypothesis it has a representation in which the vertices of the outer face form a "ladder" in the segment representation of the graph H. Since the graph is bipartite the segments representing vertices $1, 2, \ldots, d$ are either all horizontal or all vertical. Without loss of generality we may assume they all are horizontal. As depicted in Fig. 1 we can extend the segments representing $1, 2, \ldots, d$ and add a new vertical segment u in such a way that u touches all segments $1, 2, \ldots, d$. The resulting representation clearly satisfies the inductive condition and completes the proof of the theorem. □

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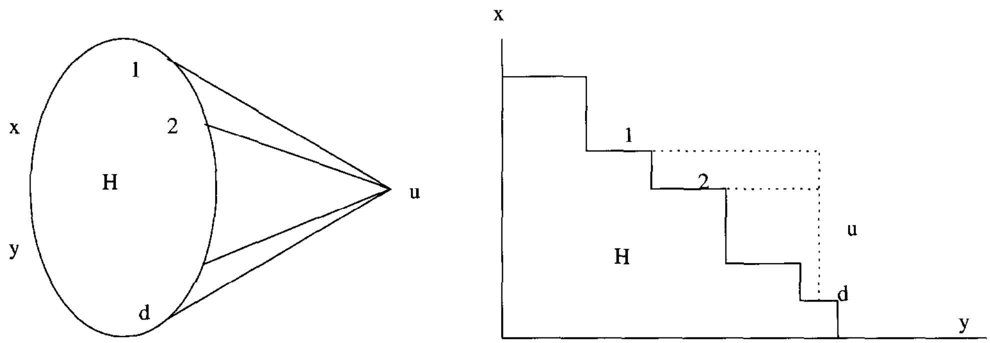

Fig. 1. Inductive construction of the ladder in the outer face of the graph G from the graph H by joining the vertex u. The vertices $1, 2, \ldots, d$ are represented by horizontal steps of the ladder.

Reference

[1] H. de Fraysseix, P.O. de Mendez, J. Pach, A left-first search algorithm for planar graphs, Discrete Comput. Geom. 13 (1995) 459–468.