Problem Set # 10

Discrete Mathematics III (Probabilistic Method)
WS 2010/11

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The solutions are due on January 18th, 14:15pm.

You are welcome to submit **at most** two neatly written exercises each week. You must achieve a total of 15 full solutions during the semester.

You should try to solve all the exercises, they are part of the final exam.

1. Let G be a graph and let P denote the probability that a random subgraph of G obtained by picking each edge of G with probability $1/2$, independently, is connected (and spanning). Let Q denote the probability that in a random 2-coloring of G, where each edge is chosen, randomly and independently, to be either red or blue, the red graph and the blue graph are both connected (and spanning). Is $Q \leq P^2$?

2. A family of subsets $\mathcal{G}$ is called *intersecting* if $G_1 \cap G_2 \neq \emptyset$ for all $G_1, G_2 \in \mathcal{G}$. Let $\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_k$ be k intersecting families of subsets of $\{1, 2, \ldots, n\}$. Prove that
$$\left| \bigcup_{i=1}^{k} \mathcal{F}_i \right| \leq 2^n - 2^{n-k}.$$
Remark: Here "union" means the usual notion of union of the families as sets and *not* the family of those sets that can be represented as unions of sets from these families.

3. Show that the probability that in the random graph $G(2k, 1/2)$ the maximum degree is at most $k-1$ is at least $1/4^k$.