Problem Set # 4

Discrete Mathematics III (Probabilistic Method)
WS 2010/11

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The solutions are due on November 16, 14:15pm.

You are welcome to submit **at most** two neatly written exercises each week. You must achieve a total of 15 full solutions during the semester.

You should try to solve all the exercises, they are part of the final exam.

1. Given two graphs G and H, the Turán number $ex(G, H)$ is the largest integer m such that there exists an H-free subgraph of G with m edges. Prove that

$$ex(G, H) \geq \left(1 - \frac{1}{\chi(H) - 1}\right) e(G).$$

2. Improve the result in Homework 1: Prove that the Ramsey number $R(4, k)$ satisfies

$$R(4, k) \geq \Omega\left(\left(\frac{k}{\ln k}\right)^2\right).$$

3. Prove that every 3-uniform hypergraph with n vertices and $m \geq n/3$ edges contains an independent set of size at least $\frac{2n^{3/2}}{3\sqrt{3}\sqrt{m}}$.

4. $(*)$ Let F be a finite collection of binary strings of finite length and assume that no two distinct concatenations of two finite sequences of codewords result in the same binary sequence. Let N_i denote the number of strings of length i in F. Prove that

$$\sum_i \frac{N_i}{2^i} \leq 1.$$